Lecture 9

In this lecture we continue our study of certain special functions. Then we introduce summation notation.

$\log_a x$

(log of x to the base a)

log: short for "logarithm"

$$\log_a x = \boxed{} \;?$$

$$a^{\boxed{}} = x$$

<u>E.g.</u>

$$\log_2 128 = \boxed{7} \;?$$

$$2^{\boxed{7}} = 128$$

$$\log_3 1 = \boxed{0} \;?$$

$$3^0 = 1$$

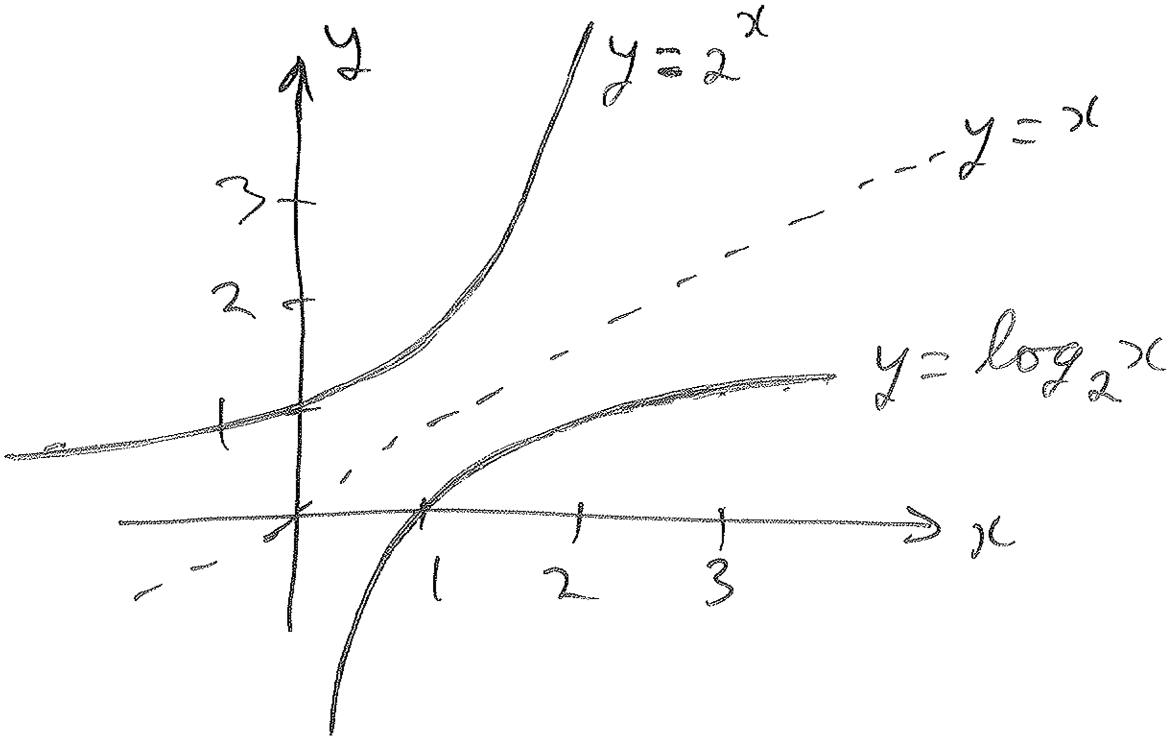
Laws of Logarithms

$$\log_a a^n = n$$

special cases
$$\begin{cases} \log_a a = 1 \\ \log_a 1 = 0 \\ \log_a a^{-1} = -1 \end{cases}$$

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a x^n = n \log_a x$$



E.g. $\log_2\left(\frac{1}{4}\right) = \ ?$

$$\log_2\left(\frac{1}{4}\right) = \log_2 4^{-1}$$
$$= -1 \cdot \underbrace{\log_2 4}_{2}$$
$$= -2$$

Useful bases

$\log x = \log_{10} x$ (many calculators)

$\ln x = \log_e x$ ($e \simeq 2.71828$)

$\lg x = \log_2 x$ (some texts)

Polynomial Functions

A function of the form

$$y = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$$

is called a <u>polynomial function.</u>

The expression on the right of the "equals" sign is a <u>polynomial.</u>

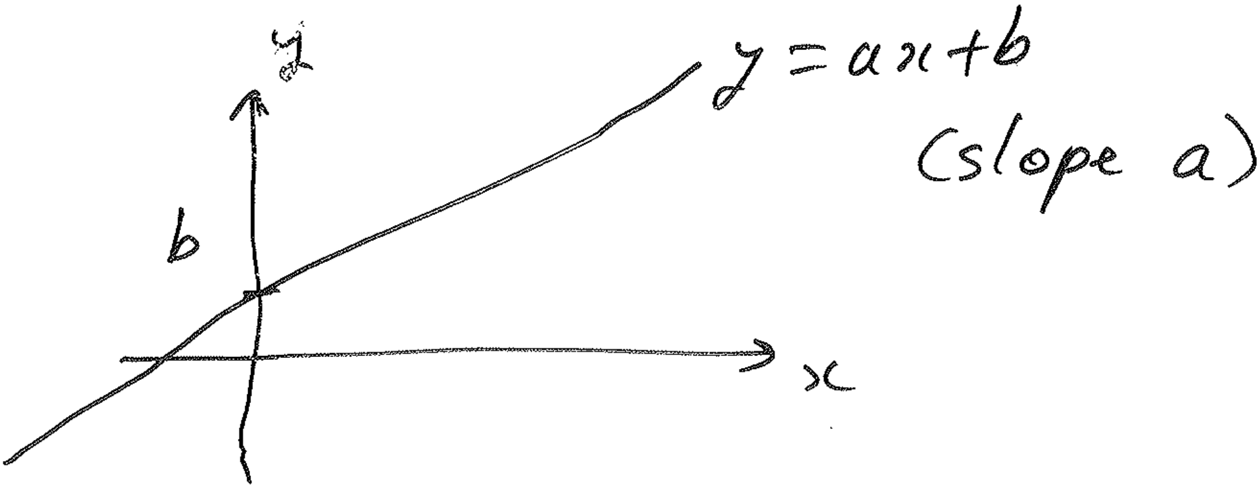
Examples

Constant functions:

$$y = a$$

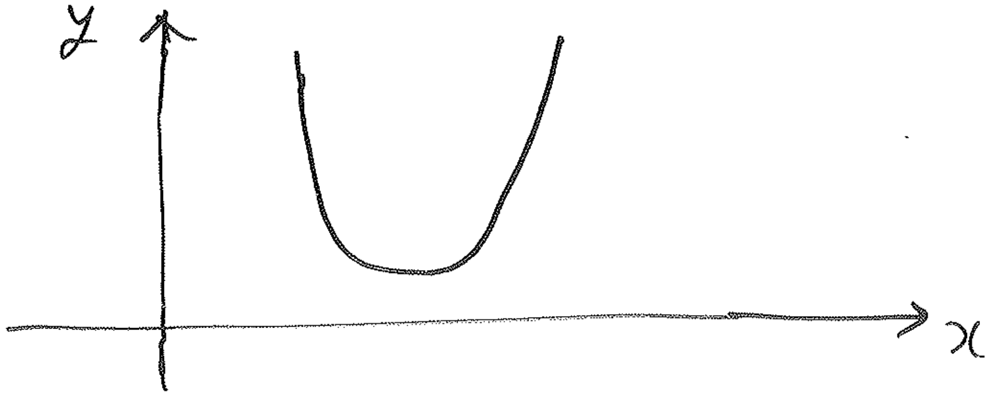
Linear functions:

$$y = ax + b$$



Quadratic functions:

$$y = ax^2 + bx + c$$



$$\underbrace{a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n}_{\text{a general polynomial}}$$

If $a_n \neq 0$, this has <u>degree</u> n.

So quadratic functions are polynomials of degree 2.

Linear functions have degree 1.

Constant functions (except $y=0$) have degree 0.

Special case: ~~for~~ technical reasons, the zero polynomial is said to have degree $-\infty$ (negative infinity) although some texts say that the degree is undefined.

Algorithms that require only polynomial time (or polynomial memory space) for implementation are of much more practical use than those of exponential or factorial complexity.

It's even better if an algorithm has logarithmic complexity.

Summation & Product Notation

$1+2+3+4+5+6+7$

$$= \sum_{i=1}^{7} i \, .$$

$\sum$ — capital sigma

In maths, this means the <u>sum</u>.

So $\sum_{i=1}^{7} i$ means "the sum of the i, from $i=1$ to $i=7$."

Similarly,

$$\sum_{i=1}^{5} 3i^2$$

means

$$3\cdot 1^2 + 3\cdot 2^2 + 3\cdot 3^2 + 3\cdot 4^2 + 3\cdot 5^2.$$

More generally, let $f(i)$ be any function.

Then

$$\sum_{i=1}^{n} f(i)$$

means

$$f(1) + f(2) + \cdots + f(n).$$

E.g. Evaluate $\displaystyle\sum_{i=0}^{4} 3^i$.

[Do as homework.]